# On Kip Thorne's limit for rotation of black holes

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## Contents



# Summary

In 1974 Kip Thorne showed that the maximum rotation of black holes cannot reach the value of M, where M means the mass of the rotational black hole of irreducible mass $M_{irr}$ and including the mass equivalent of rotational energy. Using simulation calculations he showed that this limit should be near 0.998*M. Much work was done since then to evaluate this limit more precisely. *The purpose of this paper is to derive the limit by simple calculation*.

The base for this calculation is a formula for Kerr-Newman black holes[1] and (in the background) the idea that our universe does not contain any matter at all but just is curved spacetime. Objects are black holes and collections of those. This has of course the consequence, that elementary particles should be kind of black holes too. For the electron this is sketched subsequently.

One another consequence will be the time dependency of the Lorentz factor. For the special case of a radial velocity field describing the expansion of our universe we can show that dark matter may be explained by the radial acceleration that must be added to every rotational system, i.e. galaxies or clusters, thus leading to a precision of MOND theory.

[1] which yields a Kip-Thorne-limit of $0.998_{172327561206}$

At the end I suggest a more general identity than that of Kerr-Newman, including the Lorentz-factor. For this generalized identity we can derive a more precise Kip-Thorne-limit of $0.998_{16730167236}$.

# Kip Thorne's limit: First approach

We are using Planck units in the calculations to come.

Remember the Kerr-Newman-identity for rotational, loaded black holes:

$M^2 = \frac{(4M_{irr}^2+Q^2)^2}{4(4M_{irr}^2-a^2)}$ **(Kerr-Newman)**

where $M_{irr}$ denotes the irreducible mass, Q the load, a the spin factor and M the mass $M_{irr}$ including the mass equivalent to rotation and load energy[2]. And all items have unit Planck length $l_p$.

For Q=0 one gets the Kerr identity for rotating black holes $M^2 = \frac{4M_{irr}^4}{4M_{irr}^2-a^2}$ **(Kerr)**

So together (a' instead of a in the Kerr-identity):

**(K)** $M^2 = \frac{4M_{irr}^4}{4M_{irr}^2-a'^2} = \frac{(4M_{irr}^2+Q^2)^2}{4(4M_{irr}^2-a^2)}$ **(KN)**

In the case of maximal spin factor $a'^2 = M^2$ for the Kerr case (K) we get: $M^2 = 2M_{irr}^2$ and thus for (KN) after short calculation:

$a^2 = M^2 - Q^2\left(1+\frac{Q^2}{8M_{irr}^2}\right) = M^2 - Q^2\left(1+\frac{Q^2}{4M^2}\right)$

$\rightarrow a = \pm M^* \sqrt{1-\frac{Q^2}{M^2}\left(1+\frac{Q^2}{4M^2}\right)}$

Setting $M_{irr}^2 = 1$ (smallest value in units of $l_p^2$), thus $M^2 = 2$ and $Q^2=\alpha$ (fine structure constant):

$a = \pm\sqrt{1-\frac{\alpha}{2}\left(1+\frac{\alpha}{8}\right)} * M = \pm 0{,}998_{172327561206} * M$ (at M=2). But since (K) and (KN) are homogeneous, we get for arbitrary natural number n:

$nM^2 = \frac{(4nM_{irr}^2+n\alpha)^2}{4(4nM_{irr}^2-na^2)}$ , therefore: $na^2 = nM^2 - nQ^2\left(1+\frac{nQ^2}{4nM^2}\right) = nM^2(1-\frac{Q^2}{M^2}\left(1+\frac{Q^2}{4M^2}\right))$, so Kip Thorne's limit also for big black holes.

## Falsification

None, since in the last section I shall make a proposal leading to a more precise value.

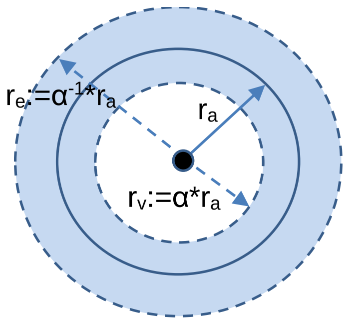
2 Note, that the term mass and load associated with M and Q is somehow misleading, since in fact we can only compare spins $M^2$ and $Q^2$. In this context, $M^2$ and $Q^2$ denote spins, that we associate with certain phenomena called gravitation and electromagnetism.

# The electron as a black hole

As already mentioned in the section *summary* the overall idea is that, what we call matter, is just a collection of black holes in a curved spacetime. Consequently the elementary particles should be black holes. Let us take a look on the candidate *electron*. A colleague once told me, that this is an old idea of Einstein. But he discarded it, because he did not trust in the existence of black holes.

The basic idea is that the electron is a black hole, circled by a photon of Compton-wavelength at radius such that spin reaches ħ (and cannot go below this value), so that this photon provides the mass m of the electron.

The radius, call it $r_a$, calculates to $r_a = \frac{h*c}{2\pi*\lambda}$ and its mass m may be derived by $E = mc^2 = h*\nu = \frac{h*c}{\lambda}$ ($\lambda=2{,}426_{3102364588}*10^{-12}$ m Compton-wavelength) to $m = \frac{h}{c*\lambda} = 9{,}109_{38356}*10^{-31}$ kg. According to interactions the electron has a uncertainty in its extent. See the following picture for illustration.




Herein α is the fine structure constant. The radius $\alpha^{-1} * r_a$ is called Bohr radius, $\alpha * r_a$ the classical electron radius, whereas $\alpha * r_a$ is a radius "inside" the electron. The small inner black circle stands for the Schwarzschild radius $r_s$. For the so-called gravitation radius $r_g$, the limit radius of a black hole of Kerr-type rotating with maximum spin we have: $r_g = \frac{1}{2} * r_s$ .

So with $m := \frac{p}{c}$, p being the amount of the photon's spin):

[in Planck units]

| | | |
|---|---|---|
| $\frac{1}{m} * r_s = \frac{2G}{c^2}$ | $\rightarrow \frac{1}{m} * r_g = \frac{G}{c^2}$ | $\frac{1}{m} * r_s = 2 \quad \rightarrow \frac{1}{m} * r_g = 1$ |
| $m * r_a = \frac{\hbar}{c}$ | | $m * r_a = 1$ |
| $m * r_e = \alpha^{-1} \frac{\hbar}{c}$ | $\rightarrow r_e = \alpha^{-1} r_a$ | $m * r_e = \alpha^{-1} \quad \rightarrow r_e = \alpha^{-1} r_a$ |
| $m * r_v = \alpha \frac{\hbar}{c}$ | $\rightarrow r_v = \alpha r_a$ | $m * r_v = \alpha \quad \rightarrow r_v = \alpha r_a$ |
| $r_s * r_a = 2 l_p^2$, | $l_p = 1{,}6_{162283732}*10^{-35}$m Planck length. | $r_s * r_a = 2$ |

These relations are also true for proton and neutron, using similar values for mass and Compton-wavelength. For the proton the values are:

$m = 1{,}6726_{2158}*10^{-27}$ kg ($r_s = 1{,}5370_{0389118943}*10^{-19}\ l_p$) and $2\pi*r_a = 1{,}3214_{098555}*10^{-15}$ m ($r_a=1{,}3012_{328800627}*10^{19}\ l_p$) $\rightarrow r_s*r_a = 2\ [l_p^2]$.

In Planck-units: Setting $Q^2 = \alpha$, $M_{irr}^2 \rightarrow \frac{1}{m} * r_g = 1$, $M^2 = r_s * r_a = 2$ and $a^2 = 2 - \alpha\left(1 + \frac{\alpha}{8}\right)$ we have the parameters of the electron as a black hole of Kerr-Newman-type.

According to this picture, one could define a *minimal lifetime* $l_e$ for the electron, meaning a timespan, in which there cannot be any interaction. This would be the timespan, the photon needs for a complete circulation. A simple calculation shows:

$l_e = \frac{2\pi r_a}{c} = \frac{h}{mc^2} = 8{,}093_{29978694393}*10^{-21}$s $= 1{,}501_{21744964211}*10^{23}$ Planck time units.

## Falsification

One could try to test, whether the minimal lifetime of the electron really exists as a limit for time to elapse between two interactions, i.e. annihilations.

# Kip Thorne's limit: Precise value

I suggest the following modification of (K)/(KN) due to the picture of a curved spacetime without matter:

**(E)** $\mathrm{M}^2 = \frac{4\mathrm{M_{irr}}^4}{4\mathrm{M_{irr}}^2 - a'^2} = \frac{4(\gamma \mathrm{M_{irr}}^2)^2}{4\mathrm{M_{irr}}^2 - \mathrm{a}^2}$ with Lorentz-factor γ. If one expands $\gamma = \frac{1}{\sqrt{1-\frac{w^2}{c^2}}}$ on $\frac{w^2}{c^2}$ into a Taylor series around the Zero point, one gets:

$$\gamma = \gamma\left(\frac{w^2}{c^2}\right) = \sum_{k=0}^{\infty} \frac{\prod_{j=0}^{k-1}(2j+1)^2}{(2k)!}\left(\frac{w^2}{c^2}\right)^k = 1 + \frac{1}{2} * \left(\frac{w^2}{c^2}\right) + \frac{3}{8} * \left(\frac{w^2}{c^2}\right)^2 + \frac{5}{16} * \left(\frac{w^2}{c^2}\right)^3 + \frac{35}{128} * \left(\frac{w^2}{c^2}\right)^4 + \cdots$$

If we stop the series after the first two summands, setting $\frac{w^2}{c^2} = \frac{Q^2}{2\mathrm{M_{irr}}^2}$, we will get: the identity (K)/(KN): $\mathrm{M}^2 = \frac{4\mathrm{M_{irr}}^4}{4\mathrm{M_{irr}}^2 - a'^2} = \frac{16\mathrm{M_{irr}}^4 * (1+\frac{Q^2}{4\mathrm{M_{irr}}^2})^2}{4(4\mathrm{M_{irr}}^2 - \mathrm{a}^2)} = \frac{(4\mathrm{M_{irr}}^2 + Q^2)^2}{4(4\mathrm{M_{irr}}^2 - \mathrm{a}^2)}$.

The picture is, that **w** is a velocity vector field, which describes the expansion of our universe, and so is radial and orthogonal on every rotating system within the universe. If we set – using Planck units - $\mathrm{M_{irr}}^2 = 1$ [$l_p^2$] und $Q^2 = \alpha * 1$ [$l_p^2$], we get: $w^2 = \frac{\alpha}{2}$. This value corresponds to the expansion of the universe of *NOW*.

**w** is not constant, but a function of time. Since expansion of our universe was nearly constant the last billions of years, one could get the impression of a constant value of α. Using $\frac{w^2}{c^2} = \frac{Q^2}{2\mathrm{M_{irr}}^2}$ we can write down general identity (E):

**(E)** $\mathrm{M}^2 = \frac{4\mathrm{M_{irr}}^4}{4\mathrm{M_{irr}}^2 - a'^2} = \frac{4(\gamma \mathrm{M_{irr}}^2)^2}{4\mathrm{M_{irr}}^2 - \mathrm{a}^2} = \frac{4\mathrm{M_{irr}}^4}{(1-\frac{Q^2}{2\mathrm{M_{irr}}^2})(4\mathrm{M_{irr}}^2 - \mathrm{a}^2)} = \frac{8\mathrm{M_{irr}}^6}{(2\mathrm{M_{irr}}^2 - Q^2)(4\mathrm{M_{irr}}^2 - \mathrm{a}^2)}$

$$\rightarrow \quad \frac{\mathrm{M}^2}{2\mathrm{M_{irr}}^2} = \frac{1}{\left(1-\frac{Q^2}{2\mathrm{M_{irr}}^2}\right)\left(2-\frac{a^2}{2\mathrm{M_{irr}}^2}\right)}$$

For Kip Thorne's limit value on the spin factor of rotational black holes we conclude from $\mathrm{M}^2 = \frac{4(\gamma \mathrm{M_{irr}}^2)^2}{4\mathrm{M_{irr}}^2 - \mathrm{a}^2}$ : $\mathrm{a} = \pm\sqrt{2 - \frac{1}{1-\frac{\alpha}{2}}} * \mathrm{M} = \pm\sqrt{\frac{1-\alpha}{1-\frac{\alpha}{2}}} * \mathrm{M}$, leading to the more precise limit value:

**(T)** $\mathrm{a} = \pm 0{,}998_{16730167236} * \mathrm{M}$ (instead of $\pm 0{,}998_{172327561206} * \mathrm{M}$).

## Falsification

One should be able to measure higher values of the fine structure constant for higher energy densities (early universe). One should also be able to fix the *more precise Kip-Thorne-limit (T)* by extended simulation calculations.

# MOND

The picture in mind, that there is only curved spacetime and matter is a collection of extreme curved spacetime, namely black holes, we take a look on the universe.

The Lorentz group is the group of linear transformations preserving the Minkowski metric. We use (+,-,-,-) as signature. The group is generated by reflections (parity and time reversal), rotations on space coordinates and boosts. The general boost may be written as

$$\mathbf{\Lambda} := \begin{pmatrix} \gamma & -\frac{\gamma}{c}\mathbf{w}^T \\ -\frac{\gamma}{c}\mathbf{w} & \mathbf{I}_3+\frac{\gamma-1}{w^2}\,\mathbf{w}\,\mathbf{w}^T \end{pmatrix}$$

**w** being a velocity vector field.
γ Lorentz-factor,
c speed of light

Note, that $\mathbf{\Lambda}$ diag(1,-1,-1,-1) $\mathbf{\Lambda}^T$ = diag(1,-1,-1,-1) is valid even for velocity fields, where **w** is time dependent. This means of course that γ gets time dependent too. Of special interest is the case of a radial velocity vector field describing the expansion of our universe.
In the identity

**(E)** $$\frac{M^2}{2M_{irr}{}^2} = \frac{1}{\left(1-\frac{Q^2}{2M_{irr}{}^2}\right)\left(2-\frac{a^2}{2M_{irr}{}^2}\right)}$$

the value of 1 may be accomplished by variations of $Q^2$ and $a^2$. Moreover, these variations may be functions of time, as long as "inverse" to each other. So again setting $\frac{w^2}{c^2} = \frac{Q^2}{2M_{irr}{}^2}$, we may assume, that both *w* and *a* may be functions of time, "inverse" to one another in the sense of identity (E) with $\frac{M^2}{2M_{irr}{}^2} = 1$.

In the environment of time=*NOW* of our universe the time derivative $\dot{w}$ may be only nearly zero. Assume that its value is $|\dot{w}(NOW)| \approx 1{,}16 \cdot 10^{-10}\,\frac{m}{s^2}$ (approximately the value of MOND theory). For every rotational system in our universe (not too far away from *NOW*) this radial acceleration (direction to the center) has to be added. If we denote *a(NOW)* the radial acceleration at radius r of the rotating system with M(r) being the mass inside the radius, G gravitational constant, this yields:

$$a(NOW,\ rotation\text{-}system) = \frac{G*M(r)}{r^2} + |\dot{w}(NOW)| = \frac{v^2}{r} \text{ with tangential velocity } \mathbf{v}.$$

$$\rightarrow v = \sqrt{\frac{G*\left(M(r) + \frac{|\dot{w}(NOW)|*r^2}{G}\right)}{r}}.$$

The term $M_v(r) := \frac{|\dot{w}(NOW)|*r^2}{G}$ has the properties of dark matter. It is neglectable for smaller values of r and it grows while distance to the center of the rotation system grows. And it ensures that no radial acceleration in this rotation system may fall below the value of ≈ $1{,}16 \cdot 10^{-10}\,\frac{m}{s^2}$, independently of the distance from the rotation system center.

Let us assume, that $\dot{w}(\boldsymbol{x})$ is a friendly function, meaning that it is differentiable on whole spacetime, evolved so far. Then $\dot{w}(t) < 0$ for a certain timespan before NOW. But that would mean that velocity of expansion of our universe would decrease in this timespan. How does this fit to the result of Perlmutter, Schmidt and Riess, that the expansion rate is increasing? See "On Doppler effects of supernovae Ia" for my doubts on this result. Second, the expansion velocity does not correspond to expansion rate but rather to the

Hubble parameter. And there are clues, that the Hubble parameter decreases (see results on Hubble parameter in the range of 72 to 74 for cosmic structures of distances around 1 billion years, and 67 to 68 for *CMB* measures extrapolated to *NOW*).

## Falsification

The **Bullet cluster** consists of two clusters of galaxies that penetrated each other leaving behind a cloud of merged gas between the now separating clusters. Although the mass concentrates in the cloud of gas the Bullet-cluster shows lensing effects primarily in the separating clusters. This is noted to be a counterexample to MOND theory and an indicator for the dark matter hypothesis. The above model states that the virtual masses add only to rotational systems but not to the cloud of gas. So the observed lensing effect may be explained without using the concept of dark matter.

Within the **Milky way** we could check tangential velocity of outer stars to be greater than the minimum tangential velocity given by a lower limit for M(r) and the virtual mass $M_V := \frac{|\dot{w}(NOW)|*r^2}{G}$. If for instance we take r = 27000 light years from the center (expected distance of the solar system) and M(r) = 80 billion solar masses $M_\odot$ *(=1.59*10*$^{41}$ *kg)*, we get:

| | |
|---|---|
| $M_V$(r) [billion $M_\odot$] | 57 |
| v(r) [km/s] | 267 |

v(r) is an actual measured value for tangential velocity of the solar system. For stars 50000 light years from the center Sagittarius A*, we would get $M_V \approx$ 195 billion solar masses. Given a good estimate of baryon mass M(r=50000ly) we should be able to compare calculated tangential velocity to measured one.

# References

Nothing special